

MAD Week 4: Iteration

Note Title

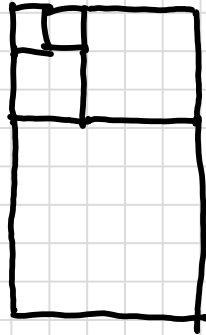
4/14/2011

Agenda/Goals

Get a good visual sense of the Fibonacci sequence

Practice iteration — both with numbers and visuals

Get a first glimpse of fractals



Short side of rectangle
1, 1, 2, 3, 5, 8, 13, 21,

1
1

1:2

or

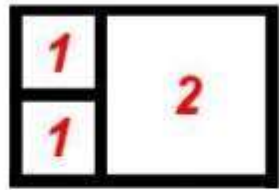
$\frac{1}{2}$

or

$\frac{2}{1}$

.5

2



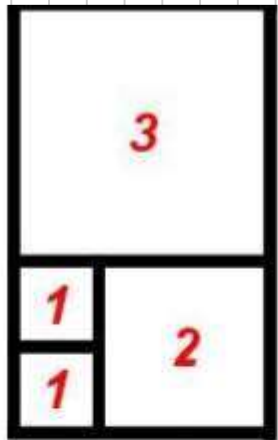
2:3

$$\frac{2}{3}$$

$$.67$$

$$\frac{3}{2}$$

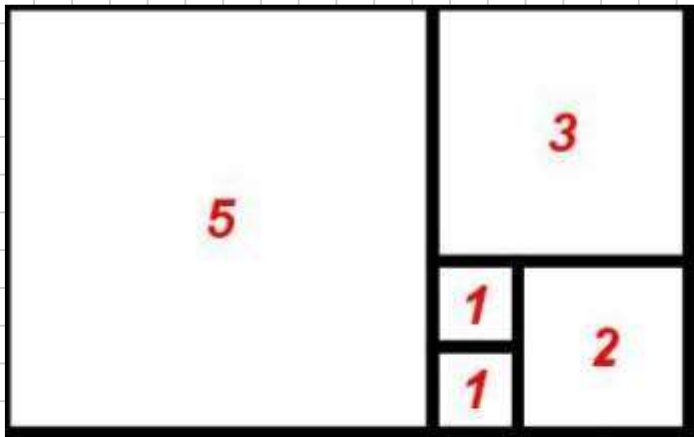
$$1.5$$



3:5

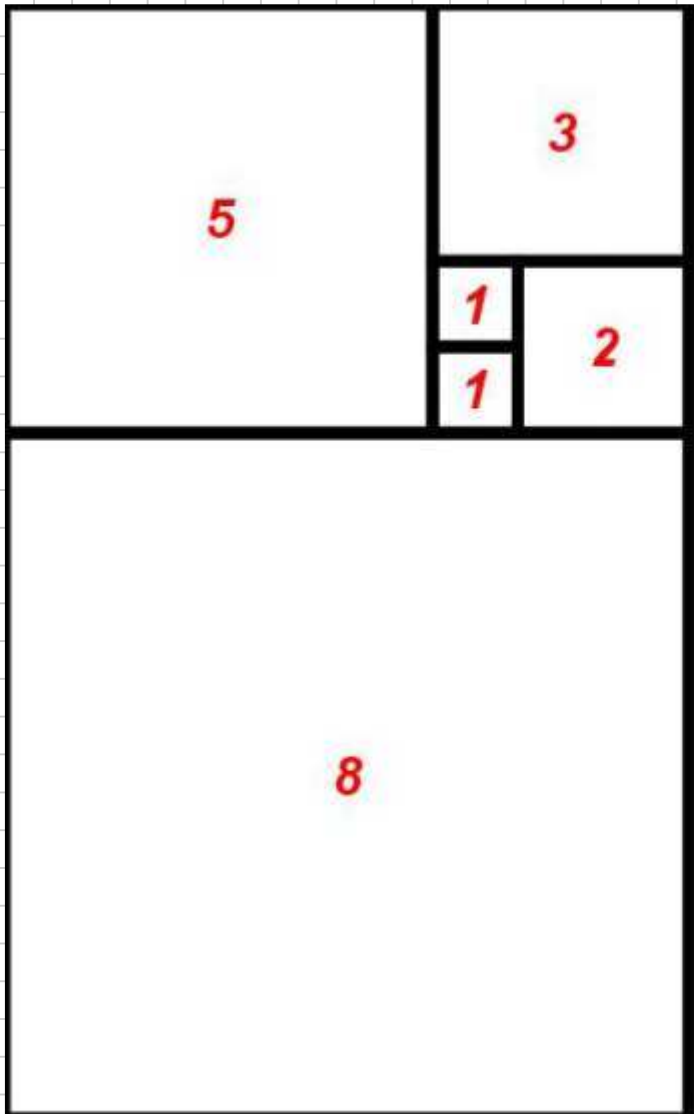
$$\frac{3}{5} \\ \cdot 6$$

$$\frac{5}{3} \\ 1.67$$



$$5:8$$

$$\begin{array}{r} 5 \\ 8 \overline{)5} \\ \underline{0} \\ .628 \end{array} \qquad \begin{array}{r} 8 \\ 5 \overline{)8} \\ \underline{5} \\ 1.6 \end{array}$$



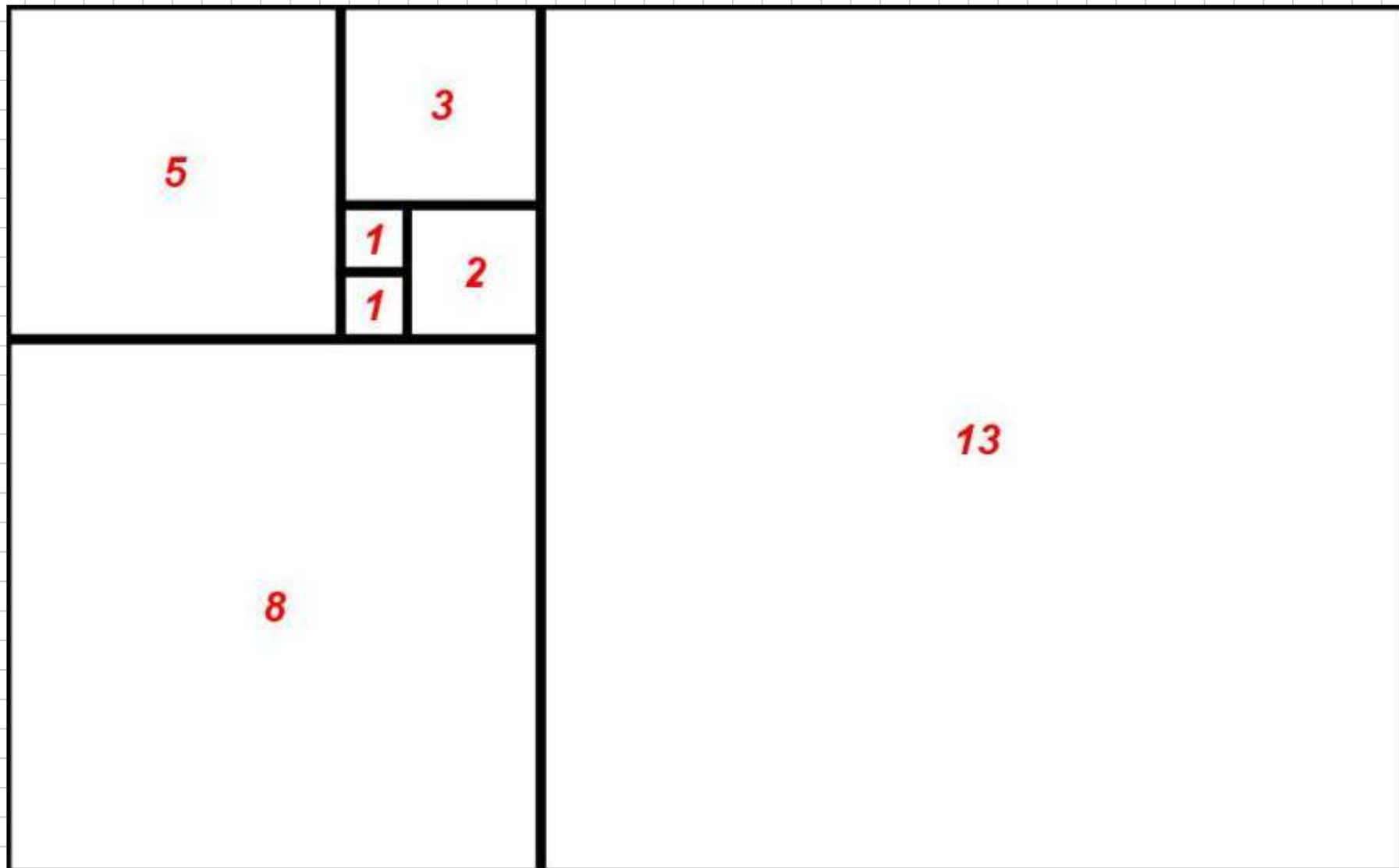
$$8:13$$

$$\frac{8}{13}$$

$$.615$$

$$\frac{13}{8}$$

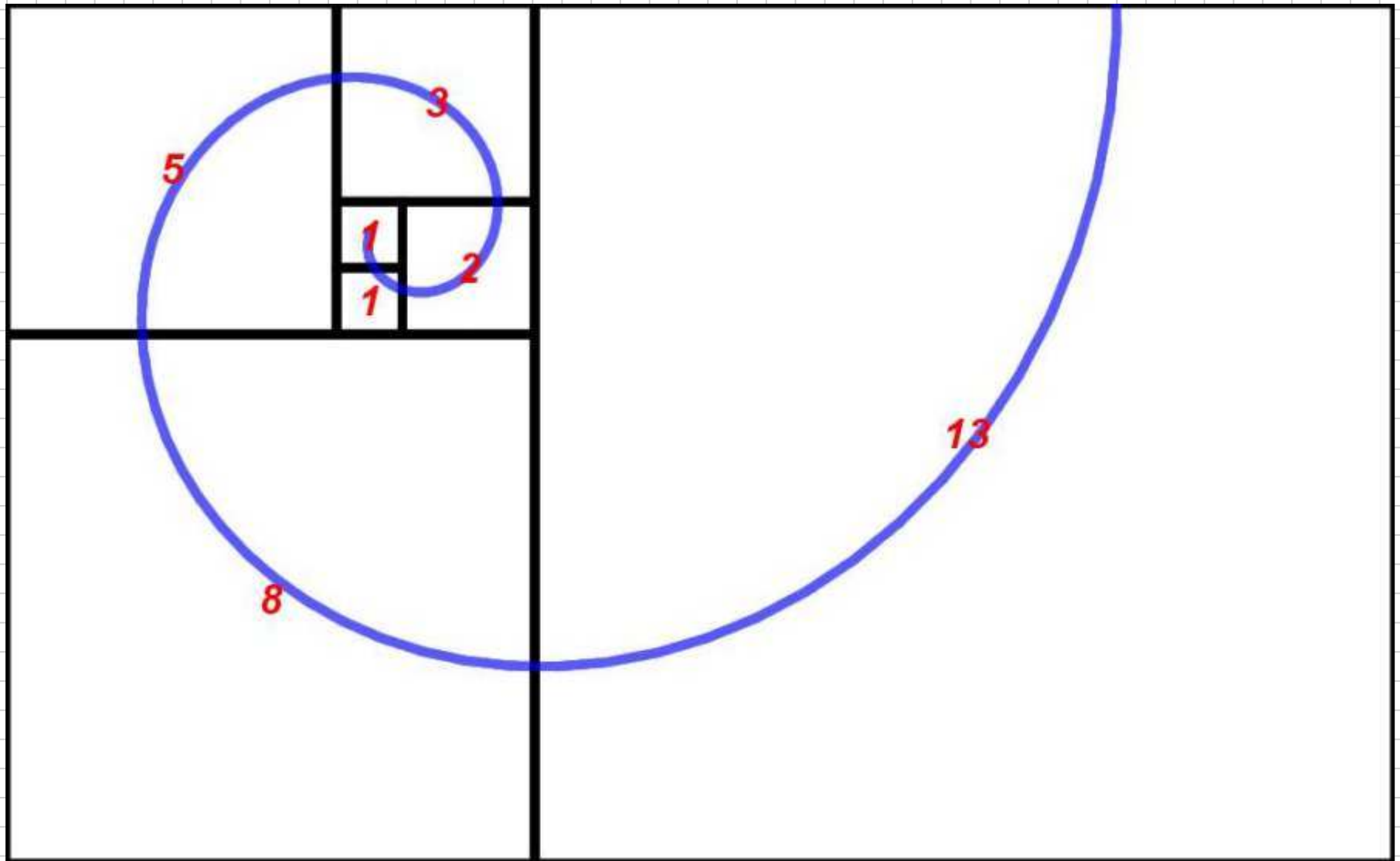
$$1.625$$



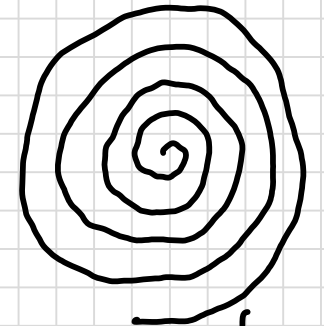
$$13:21$$

$$\frac{13}{21} = .619$$

$$\frac{21}{13} = 1.616$$



This is a kind of spiral known as a logarithmic spiral -

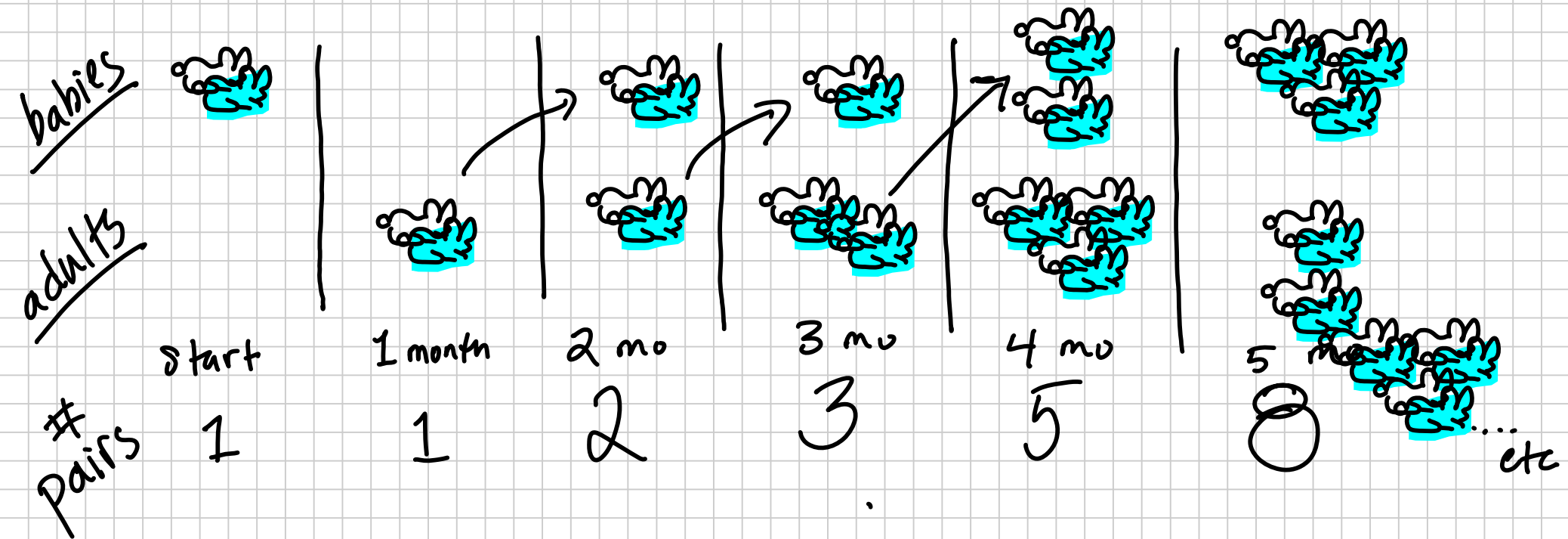


Archimedean spiral

This is our first fractal - it looks the same as we move out (and the pure mathematical creature is the same at any scale)

"Thought Experiment": Rabbit Breeding

- Start with two newborn rabbits
- Takes one month for newborn pair to mature, then they start breeding
- Adult female has 1 M/F pair of babies each month
- Rabbits live forever!



Fibonacci Sequence

1
1

← seeds

$$1+1=2$$

$$1+2=3$$

$$2+3=5$$

$$3+5=8$$

$$5+8=13$$

$$8+13=21$$

$$21+13=34$$

etc ...

rule: add two previous numbers to get next number

ITERATION

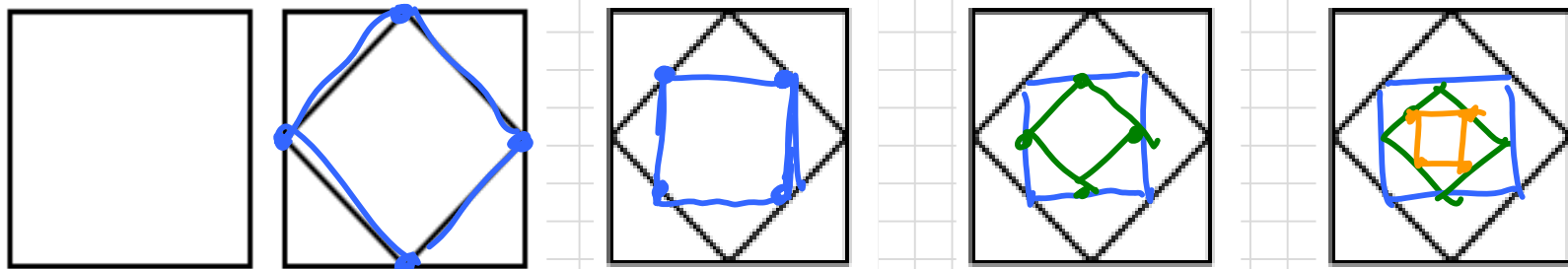
Iteration

The act of repeating a process, where the results of one step ("iteration") are used as a starting point for the next step.

The fibonacci sequence is an example of iteration

1, 1, $1+1=2$, $2+1=3$, $3+2=5$, etc ...

Another example of iteration:

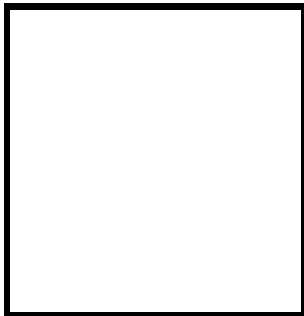


Stage: 0 1 2 3

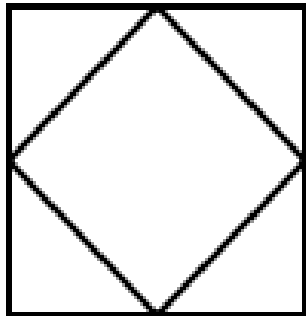
Iterative step: Connect midpoints of sides of smallest square

Continuing to iterate forever produces a limit pattern:

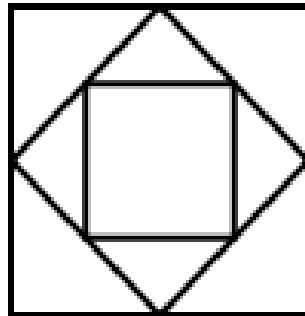
Iteration #0
(the initiator)



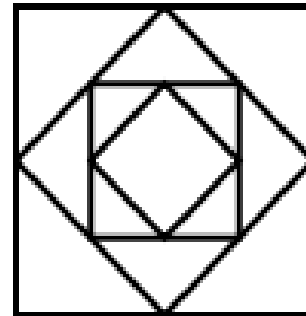
Iteration #1



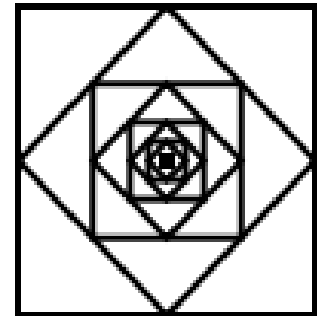
Iteration #2



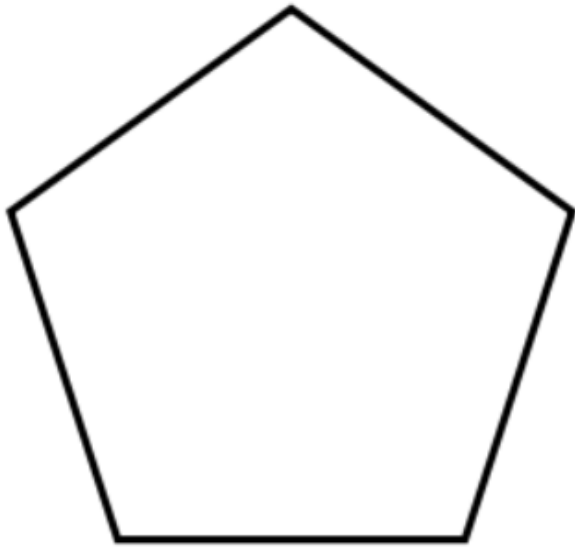
Iteration #3



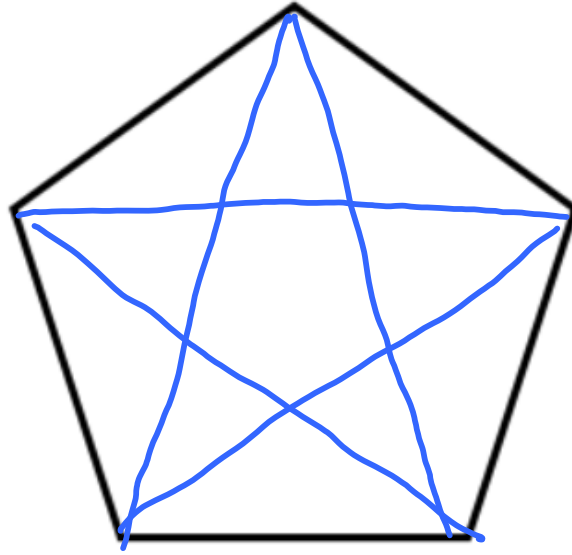
... Limit



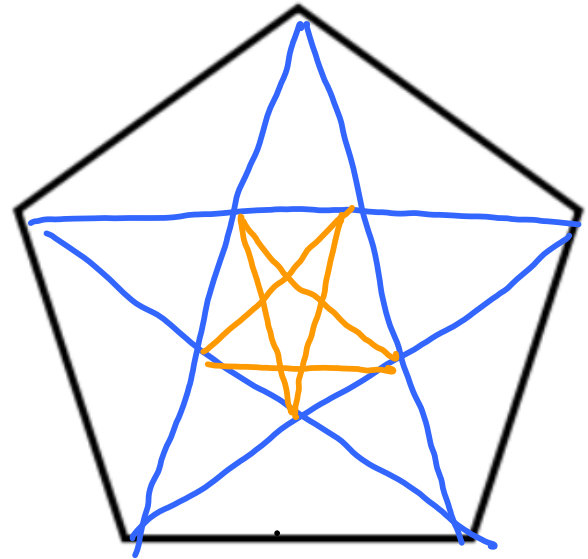
This limit is
a fractal



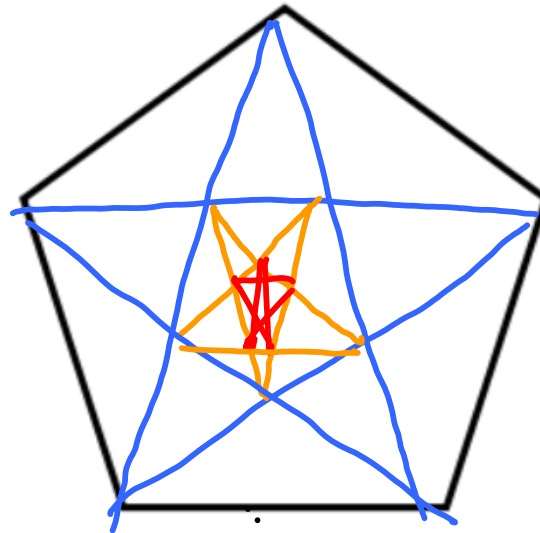
Iteration #0
(Start!)



Iteration #1



Iteration #2



Iteration #3